Intervalles de confiance pour la prime de réassurance en présence de risques extrêmes

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Résumé. Nous nous intéressons à la construction d'intervalles de confiance pour la prime de réassurance en présence de risques extrêmes. Une première méthode de construction d'intervalles de confiance, simple à mettre en œuvre, repose sur la normalité asymptotique d'un estimateur de la prime, proposé par Necir et al. (2007). Nos simulations sur des échantillons de taille finie montrent néanmoins que la probabilité de couverture de ces intervalles peut être très inférieure au niveau nominal. Dans cet exposé, nous proposons donc deux autres méthodes de construction d'intervalles de confiance pour la prime de réassurance en présence de risques exrêmes. La première est basée sur un rapport de vraisemblance, la seconde sur une méthode de pondération des observations dans un calcul de vraisemblance sous contraintes. Nous en déduisons deux variables asymptotiquement pivotales, qui nous permettent de construire des intervalles de confiance asymptotiques. Ces deux méthodes, ainsi que la méthode basée sur l'estimateur de Necir et al. (2007), sont évaluées par simulations puis illustrées sur un jeu de données contenant les montants de sinistres incendie. La méthode basée sur la pondération des observations dans un calcul de vraisemblance sous contraintes apparaît comme la plus performante, en termes de probabilité de couverture et de longueur des intervalles.

Mots-clés. Estimateur de Hill, rapport de vraisemblance, méthode data tilting

Abstract. We consider the construction of confidence intervals for the reinsurance premium of extreme risks. A straightforward method is based on the asymptotic normality of an estimator of the premium proposed by Necir et al. (2007). However, our simulations suggest that the coverage accuracy of the resulting intervals can be quite far from the nominal confidence level. Therefore in this talk, we propose two alternative methods. One is based on a likelihood ratio, the other on a data tilting method (data tilting amounts to a constrained minimisation problem of some distance function). These methods and the method based on the asymptotic normality of Necir et al. (2007) estimator are evaluated via simulations and illustrated on a real data set of fire loss. The data tilting method appears to be the most efficient, in terms of coverage probabilities and intervals length.

Keywords. Hill estimator, likelihood ratio method, data tilting method

1 Objectives

Extreme events, which arise in a wide variety of domains (e.g., environment, industry, finance) can cause considerable loss in insurers portfolio. It is thus crucial for (re)-insurance companies to determine adequate premiums for extreme risks.
One common premium calculation principle, due to Wang (1996), is based on a distortion function, that is, an increasing and concave function $g : [0, 1] \to [0, 1]$ such that $g(0) = 0$ and $g(1) = 1$. If X is a random risk with distribution function F, the distortion risk measure based on g is defined as

$$\Pi(g) = \int_0^\infty g(1 - F(x))\, dx.$$

Several distorsion functions have been proposed. Letting $g(x) = x^{1/\rho}$, for $\rho \geq 1$, yields the popular Proportional Hazard (PH) transform. The PH premium of X is given by:

$$\Pi_\rho = \int_0^\infty (1 - F(x))^{\frac{1}{\rho}}\, dx,$$

which can be seen as a distorted expectation of X (the parameter ρ controls the amount of risk loading in the premium, and is called the risk aversion index).
Now, in reinsurance, a standard practice is that the reinsurer compensates the cedant's loss only above a certain retention amount $R > 0$. In this case, the reinsurer will not pay the insurer if X is less than or equal to R and will pay $(X - R)$ if X exceeds R. The amount paid by the reinsurer is thus $(X - R)_+$, where $x_+ = \max(0, x)$, and the corresponding PH premium for the layer $[R, \infty)$ is defined as the distorted expectation of $(X - R)_+$:

$$\Pi_{\rho,R} = \int_R^\infty (1 - F(x))^{\frac{1}{\rho}}\, dx.$$

Various estimates of $\Pi_{\rho,R}$ have been proposed for heavy-tailed insured risk, see for example Necir and Boukhetala (2004), Necir et al. (2007), Benkhelifa (2014), Ahmedou et al. (2023).

Our objective, here, is to construct confidence intervals for $\Pi_{\rho,R}$. In Necir et al. (2007), an asymptotically normal estimate of $\Pi_{\rho,R}$ is proposed. From this, it is straigthforward to construct a confidence interval for $\Pi_{\rho,R}$. However, our simulation study (Ameraoui et al., 2023) suggests that the coverage probabilities of the resulting intervals can be quite far from the nominal confidence level. Thus, in this work, we consider two alternative methods, namely: *i*) a likelihood ratio method, and *ii*) a data tilting method. Both have already proved useful to construct confidence intervals for the tail index and high quantiles of a heavy-tailed distribution, see for example Lu and Peng (2002), Peng and Qi (2006), Tursunalieva and Silvapulle (2016). We adapt them to the setting of interval estimation of the PH premium under high-excess loss layer.

2 Results

We propose two asymptotically pivotal functions for $\Pi_{\rho,R}$, based on the likelihood ratio and data tilting methods, and we prove their convergence in distribution to a χ^2_1 distribution.

From these results, we can construct confidence intervals for $\Pi_{\rho,R}$. Their performance are assessed in a simulation study. Evaluation criteria include coverage probabilities and interval length.
Our results (see Ameraoui et al., 2023) suggest that the data tilting method provides the best results. The confidence intervals based on this method have a coverage accuracy close to the nominal level, and their length are generally smaller. They are also less sensitive to the choice of the sample fraction used to calculate the various quantities involved in the intervals (such as the Hill estimate of the tail index of F).

Bibliographie

Ahmedou, S., Deme, E.H. and Fofana, S. (2023), An improved estimator of distortion risk premiums under dependence insured risks with heavy-tailed marginals, *Far East Journal of Theoretical Statistics*, 67(3), pp. 243-277.

Ameraoui, A., Boukhetala, K. and Dupuy, J.-F. (2023), Confidence intervals for the proportional hazard reinsurance premium for heavy-tailed claims, submitted.

Benkhelifa, L. (2014), Kernel-type estimator of the reinsurance premium for heavy-tailed loss distributions, *Insurance: Mathematics and Economics* 59, pp. 65-70.

Lu, J.C. and Peng, L. (2002), Likelihood based confidence intervals for the tail index, *Extremes* 5, pp. 337-352.

Necir, A. and Boukhetala, K. (2004), Estimating the risk adjusted premium of the largest reinsurance covers, *In: Antoch, Jaromir (Ed.), Proceeding of Computational Statistics. Physica-Verlag, Springer*, pp. 1577-1584.

Necir, A., Meraghni, D. and Meddi, F. (2007), Statistical estimate of the proportional hazard premium of loss, *Scandinavian Actuarial Journal*, 3, pp. 147-161.

Peng, L. and Qi, Y. (2006), Confidence regions for high quantiles of a heavy tailed distribution, *The Annals of Statistics*, 34(4), pp. 1964-1986.

Tursunalieva, A. and Silvapulle, P. (2016), Nonparametric estimation of operational value-at-risk (OpVaR), *Insurance: Mathematics and Economics*, 69, pp. 194-201.

Wang, S. (1996), Premium calculation by transforming the layer premium density, *Astin Bulletin*, 26(1), pp. 71-92.