# OT and EOT QQ-plots. Application in Risk Analysis and Management

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**Résumé**

Univariate Q-Q plot is a very powerful visualisation tool, used to compare two distributions. Turning to multivariate quantiles, various approaches are possible. We refer the reader to Serfling (2002), Chaudhury (1996), Singha et al. (2023-24), Singha (2024) and references therein for an extensive survey on multivariate quantiles. Due to the absence of natural ordering, there is no straightforward extension of QQ plots for multivariate samples, as there is no natural extension of quantile function for multivariate distributions.
Using the geometric multivariate quantiles developed by Chaudhury (1996) and their property of unique characterization of the underlying distribution, Dhar et al. (2014) constructed component wise QQ plots for comparing multivariate distributions. Considering a similar approach as Easton and McCulloch (1990) and Dhar et al. (2014), Singha et al. extended this graphical tool when using optimal transport (OT) map and optimal potential (OP) function, referred as OT QQ plot and OP QQ plot, respectively. It was also shown that, as the size of the samples increases, the Q-Q plots become arbitrarily close to the straight line passing through the origin and with slope 1 if and only if the samples are drawn from the same distribution.

In order to generate an OT Q-Q plot, one must first calculate empirical OT maps. However, computing empirical OT maps can be costly, especially when size of the sample is large. Practical solutions have been proposed in the literature, one of the most popular approaches being entropy regularization (Cuturi, 2013). By selecting the regularization parameter to be sufficiently small, the entropy regularized map (EOT) can closely approximate the OT map. Moreover, the EOT also characterizes a distribution uniquely (see Singha, 2024), justifying the construction of OT and EOT QQ-plots. As for geometric quantiles (see Dhar et al., 2014), test statistics for comparing two distributions based on the proposed Q-Q plots, can also be developed to assess their relevance.

Turning to applications, we show the attractiveness of (E)OT QQ plots to develop stress scenarios for risk management purpose. This approach should provide regulators and risk managers with a robust tool to identify the most suitable extreme scenarios for targeted stress testing at any specified probability level.

This presentation is based on joint works with S. Singha (TIFR-CAM), S. Vadlamani (TIFR-CAM) and M. Dacorogna (PRS).

References:

*Intervenant

Cuturi, M. (2013). Sinkhorn distances: Lightspeed computation of optimal transportation distances. *Advances in Neural Information Processing Systems* 26, 2292–2300

Dhar, S., Chakraborty, B. and Chaudhuri, P. (2014). Comparison of multivariate distributions using quantile–quantile plots and related tests. *Bernoulli* 20(3), 1484–1506

Easton, G.S. and McCulloch, R.E. (1990). A multivariate generalization of quantile-quantile plots. *Journal of the American Statistical Association* 85, 376–386

Serfling, R. (2002). Quantile functions for multivariate analysis: Approaches and applications. Statistica Neerlandica 56, 214–232

Singha, S. (2024). Characterising distributions and their tails using multivariate quantiles and depths. *Doctoral Thesis in Mathematics*, TIFR
Singha, S., Kratz, M., Vadlamani, S. (2023-24). Extremal behaviour and convergence rates for sample–based geometric quantiles and half space depths. *ArXiv:2023-0610789v2*

**Mots-Clés:** Multivariate quantiles, Optimal Transport (OT) Map, Entropy OT (EOT), QQ plots, Quantitative risk analysis, Stress scenarios, Test statistics